MULTIMEDIA UNIVERSITY

STUDENT ID NO

MULTIMEDIA UNIVERSITY

FINAL EXAMINATION

TRIMESTER 2, 2017/2018

DIM5068 – MATHEMATICAL TECHNIQUES 2
(For DIT students only)

14 MARCH 2018
09.00 am - 11.00 am
(2 Hours)

INSTRUCTIONS TO STUDENT

1. This question paper consists of **2** pages with **4** questions. Key formulae are given in the Appendix.
2. Answer **ALL** questions.
3. Write your answers in the answer booklet provided.
4. All necessary working steps must be shown.

Question 1

a) Differentiate the following functions with respect to x by using **Chain Rule.**

i) $y = \sin(x^4 - 6x^2 - 2x)$. (5 marks)

ii) $y = -\dfrac{4}{\sqrt[3]{3x^2 + 5x}}$. (6 marks)

b) Differentiate $9y^2 - 5x^4 + 3xy^2 = x^3y^3$ by using implicit differentiation. (7 marks)

c) Find the coordinates of the critical points of the given function $f(x) = x^3 - x^2 - x$. (7 marks)

[TOTAL 25 MARKS]

Question 2

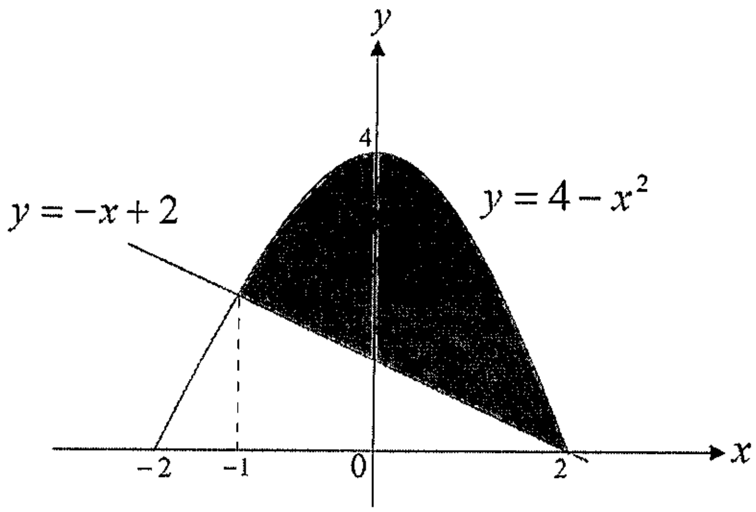
a) Evaluate the following definite integral.

i) $\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} (2y^3 + \cos y)\, dy$. [Note: leave your answer in terms of π] (5 marks)

ii) $\int_0^{\pi} (3x+5)\sin x\, dx$. [Hint: use **Integration by Parts**] (8 marks)

b) Find the indefinite integral $\int (3x^4 - 7)^6 (12x^3)\, dx$.
[Hint: use **Substitution Method**] (5 marks)

c) Find the area of the region bounded by the graphs of $y = 4 - x^2, y = -x + 2, x = -1, x = 2$ as shown below. (7 marks)



[TOTAL 25 MARKS]

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Question 3

a) Solve the first order differential equation $\frac{dy}{dx} = \frac{x^3 + 6x + 2}{y}$, by using the **Separable Method**. (5 marks)

b) Given the differential equation $\frac{dy}{dx} + \frac{y}{x} = \frac{29}{x}$.

  i) Identify the $p(x)$ and $q(x)$. (2 marks)

  ii) Calculate the integrating factor, μ. (1.5 marks)

  iii) Find y given $\mu y = \int \mu q(x)\,dx$. (3.5 marks)

  iv) From your answer in part b(iii), determine the solution of the initial value problem if $y(7) = 33$. (2 marks)

  v) State the general solution of y. (1 mark)

c) Given the non-homogeneous differential equation $8y'' - 6y' + y = 29x + 4$.

  i) Determine the complementary solution, y_c. (3 marks)

  ii) Compute the particular solution, y_p. (7 marks)

[TOTAL 25 MARKS]

Question 4

a) Let $\mathbf{a} = 2\mathbf{i} + 4\mathbf{j} - 2\mathbf{k}$ and $\mathbf{b} = 7\mathbf{i} + 4\mathbf{j} + 5\mathbf{k}$.

  i) Compute $3\mathbf{b} \cdot (-2\mathbf{a})$. (4 marks)

  ii) Find the value of x and y if $\mathbf{a} + \mathbf{b} = \langle 9, y + 3x, x \rangle$. (3 marks)

b) Given that $|\mathbf{u}| = 5, |\mathbf{v}| = 7$, $\mathbf{u} \cdot \mathbf{v} = 3x - 15$ and angle between $\mathbf{u}$ and $\mathbf{v}$ is $\frac{\pi}{2}$, find x. (3 marks)

c) Anita wants to design a card with a triangular shape. Given the vertices of the triangle $P = (2, 9, 0)$, $Q = (-4, 1, 9)$, and $R = (8, -7, 0)$.

  i) Determine $\overrightarrow{PQ}$ and $\overrightarrow{PR}$. (2 marks)

  ii) Calculate the cross product of $\overrightarrow{PQ}$ and $\overrightarrow{PR}$. (3 marks)

  iii) Compute the total area of the card. Round your answer to 2 decimal points. (3 marks)

d) If a line passing through the points (6, 6, 2) and (7, −5, 5), compute;

  i) The parametric equations of the line. (4 marks)

  ii) The symmetric equations of the line. (3 marks)

[TOTAL 25 MARKS]

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APPENDIX

Derivatives: $f'(x)=\lim\limits_{h\to 0}\dfrac{f(x+h)-f(x)}{h}$

Differentiation Rules

General Formulae

1. $\dfrac{d}{dx}[f(x)g(x)]=f(x)g'(x)+g(x)f'(x)$
2. $\dfrac{d}{dx}\left[\dfrac{f(x)}{g(x)}\right]=\dfrac{g(x)f'(x)-f(x)g'(x)}{[g(x)]^2}$
3. $\dfrac{d}{dx}(x^n)=nx^{n-1}$
4. $\dfrac{d}{dx}[f(u)]=\dfrac{dy}{du}\cdot\dfrac{du}{dx}$

Exponential and Logarithmic Functions

1. $\dfrac{d}{dx}(e^x)=e^x$
2. $\dfrac{d}{dx}(a^x)=a^x\ln a$
3. $\dfrac{d}{dx}(\ln x)=\dfrac{1}{x}$
4. $\dfrac{d}{dx}(\log_a x)=\dfrac{1}{x\ln a}$

Trigonometric Functions

1. $\dfrac{d}{dx}(\sin x)=\cos x$
2. $\dfrac{d}{dx}(\cos x)=-\sin x$
3. $\dfrac{d}{dx}(\tan x)=\sec^2 x$
4. $\dfrac{d}{dx}(\csc x)=-\csc x\cot x$
5. $\dfrac{d}{dx}(\sec x)=\sec x\tan x$
6. $\dfrac{d}{dx}(\cot x)=-\csc^2 x$

Table of Integrals

1. $\int u\,dv=uv-\int v\,du$
2. $\int u^n\,du=\dfrac{u^{n+1}}{n+1}+C,\quad n\neq -1$
3. $\int\dfrac{du}{u}=\ln|u|+C$
4. $\int e^u du=e^u+C$
5. $\int \sin u\,du=-\cos u+C$
6. $\int \cos u\,du=\sin u+C$
7. $\int \sec^2 u\,du=\tan u+C$
8. $\int \csc^2 u\,du=-\cot u+C$
9. $\int \sec u\tan u\,du=\sec u+C$
10. $\int \csc u\cot u\,du=-\csc u+C$

Application of Integrals:

Areas between Curve, $A=\int_a^b [f(x)-g(x)]\,dx$

Differential Equations

Linear Differential Equations

$\frac{dy}{dx} + p(x)y = q(x) \quad \Rightarrow \quad \mu y = \int \mu q(x)\,dx$, where $\mu = e^{\int p(x)\,dx}$

Constant Coefficient of Homogeneous Equations

Roots of Auxiliary Equation, $r = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

General Solutions to the Auxiliary Equation:

2 Real & Unequal Roots ($b^2 - 4ac > 0$)	$y = c_1 e^{r_1 x} + c_2 e^{r_2 x}$
Repeated Roots ($b^2 - 4ac = 0$)	$y = c_1 e^{rx} + c_2 x e^{rx}$
2 Complex Roots ($b^2 - 4ac < 0$)	$y = e^{ax}(c_1 \cos bx + c_2 \sin bx)$

Constant Coefficient of Non-Homogeneous Equations

$y = y_c + y_p$ [y_c: complementary solution, y_p: particular solution]

Vector

Length of Vector

The length of the vector $\mathbf{a} = \langle a_1, a_2, a_3 \rangle$ is $|\mathbf{a}| = \sqrt{a_1^2 + a_2^2 + a_3^2}$.

Dot Product

If θ is the angle between the vector $\mathbf{a} = \langle a_1, a_2, a_3 \rangle$ and $\mathbf{b} = \langle b_1, b_2, b_3 \rangle$, then

$\mathbf{a} \cdot \mathbf{b} = a_1 b_1 + a_2 b_2 + a_3 b_3 = |\mathbf{a}||\mathbf{b}| \cos\theta$

Cross Product

If θ is the angle between the vector $\mathbf{a} = \langle a_1, a_2, a_3 \rangle$ and $\mathbf{b} = \langle b_1, b_2, b_3 \rangle$, then

$\mathbf{a} \times \mathbf{b} = \langle a_2 b_3 - a_3 b_2, a_3 b_1 - a_1 b_3, a_1 b_2 - a_2 b_1 \rangle$

$|\mathbf{a} \times \mathbf{b}| = |\mathbf{a}||\mathbf{b}| \sin\theta$

Area for parallelogram *PQRS*

$= \left|\vec{PQ} \times \vec{PR}\right|$

Area for triangle PQR

$= \frac{1}{2}\left|\vec{PQ} \times \vec{PR}\right|$

Equation of Lines

Vector equation: $\mathbf{r} = \mathbf{r}_0 + t\mathbf{v}$

Parametric equations: $x = x_0 + at \qquad y = y_0 + bt \qquad z = z_0 + ct$

Symmetric equation: $\frac{x - x_0}{a} = \frac{y - y_0}{b} = \frac{z - z_0}{c}$

Equation of Planes

Vector equation: $\mathbf{n} \cdot \mathbf{r} = \mathbf{n} \cdot \mathbf{r}_0$

Scalar equations: $a(x - x_0) + b(y - y_0) + c(z - z_0) = 0$

Linear equation: $ax + by + cz + d = 0$

Angle between Two Planes: $\theta = \cos^{-1}\left(\frac{\mathrm{n}_1 \bullet \mathrm{n}_2}{|\mathrm{n}_1||\mathrm{n}_2|}\right)$